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IFAC PapersOnLine 52-6 (2019) 31–38

Optimal Control Strategy for Relief Supply Considering Information and Demand Uncertainty after a Major Disaster ⋆

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Abstract: Humanitarian logistics is important for minimizing the damage after a disaster. In Japan, based on past disasters, three empirical control strategies related to humanitarian logistics have been proposed: two relief transportation strategies, and an information strategy without ICT. This paper reveals the mathematical properties of these empirical strategies using an analytic model with closed-form solution. Our approach is based on the stochastic optimal control theory that has never been applied for analyzing humanitarian logistics. Specifically, we formulate the inventory distribution problem considering demand uncertainty as a stochastic optimal control problem with the objectives of minimizing inventory holding and handling costs. Additionally, we consider information uncertainty after a disaster using the Bayesian updating process. This process, by updating at different intervals among depots, models information asynchrony caused by not using ICT. Finally, we analyze the optimal control strategy to reveal the mathematical properties of three empirical strategies. Our results clarify that the two empirical transportation strategies are effective. However, we suggest that in the empirical information strategy without ICT the information paradox, wherein the system gets worse by using information, may occur.

Keywords: Optimal control, Stochastic control, Information systems, Humanitarian logistics, Bullwhip effect

1. INTRODUCTION

Given the frequency of major disasters around the world, humanitarian logistics is garnering attention. For example, the Cabinet Office in Japan (2018) reported that the Kumamoto Earthquakes, which occurred on April 14 and 16, 2016 and had a maximum seismic intensity of 7, caused 267 fatalities and destroyed, at least partially, around 200,000 houses. Furthermore, the Fire and Disaster Management Agency (2018) reported that over 200 of them were disaster-related deaths that could have been avoided by the timely arrival of suitable relief goods. The aim of humanitarian logistics is to achieve the 6Rs: the Right product, in the Right quantity, at the Right place, at the Right time, at the Right cost, and in the Right condition (Chomilier (2010)), thereby minimizing such disaster-related deaths.

To ensure the success of humanitarian logistics, the Cabinet Office in Japan included two control strategies in its disaster management plan: feedback control called the pull-mode support, where relief goods are transported in response to requests from shelters, and sequence control called the push-mode support, where their need is predicted and they are transported before requests. In the 2011 Great East Japan Earthquake all the strategies were the pull-mode. However, affected local governments usually take time to obtain accurate information after a major disaster, so the push-mode has also been planned and was implemented for the first time in the Kumamoto

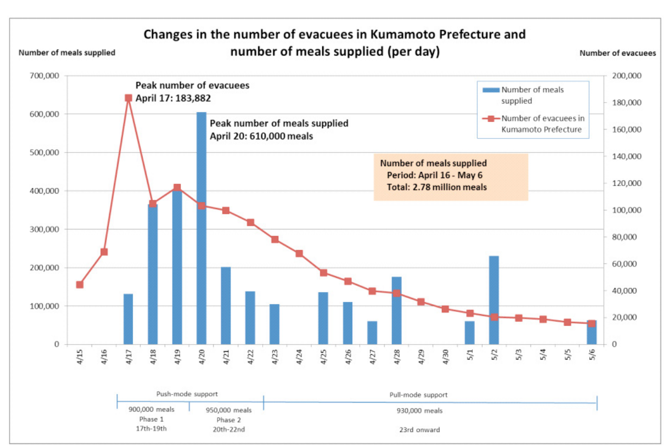

Fig. 1. Number of evacuees in Kumamoto Prefecture and number of meals supplied (Cabinet Office in Japan (2017))

⋆ This study was supported by JSPS Grant-in-aid (KAKENHI) #16H02368.

2405-8963
Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2019.08.144

Earthquake. In these control strategies, the supply chain network (SC network) involves two stages: the first stage is transportation from primary depots outside the affected area to secondary depots inside the affected area and the second stage is transportation from secondary depots to shelters, called last mile transportation. Although many major earthquakes have occurred in Japan, Holguín-Veras et al. (2014) and The Committee of Infrastructure Planning and Management (2016) reported that secondary depots, last mile transportation, and information flow became bottlenecks because of the breakdown of depots, road networks, and communication infrastructure. Furthermore, these control strategies have not been effective in past disasters. For example, Cabinet Office in Japan (2017) reported that the peak number of evacuees and that of meals supplied were different in Kumamoto earthquake (Fig.1).

Based on past disasters, new empirical control strategies were proposed. Higuchi (2017) and Ito et al. (2017) proposed the push-mode, where relief goods are transported directly from primary depots to shelters and secondary depots are abandoned, based on the success cases of Fukuoka City (2016) in the Kumamoto earthquake. Kubo and Hashimoto (2016) showed that an offline transmission system without ICT (e.g. the Kanban System using offline cards to signal demand step by step) is effective for the pull-mode. However, these strategies are empirical and might not always be effective in future disasters, although they may be effective in disasters that bear similarity to past disasters. In order to propose control strategies for future disasters, it is essential to develop an inventory distribution model.

Conventional inventory distribution models can provide optimal inventories and supplies using heuristic techniques, because most of these problems belong to the NP class. In terms of information, these are classified into two types: the offline model (e.g. Beamon and Kotleba (2006) and Barbarosoğlu and Arda (2004)), where only prior information is input, and the online model (e.g. Jaillet et al. (2002) and Sheu (2010)), where information collected after a disaster is input. The former corresponds to the push-mode modeling, which calculates strategies in advance based on predicted values. However, it is insufficient to evaluate only the push-mode because the control strategy may switch from a push-mode to a pull-mode as information becomes available. By contrast, in the latter, it is possible to evaluate the pull-mode in terms of utilizing requests from the shelters. Conventional online models assume information sharing using ICT, which contradicts the proposal by Kubo and Hashimoto (2016). When ICT isn't used, information asynchrony occurs in the SC network and each depot has different subjective information. Moreover, in such cases, Chen et al. (2000) revealed that the bullwhip effect occurs, in that uncertainty increases as information propagates upstream of the SC network. After a disaster, there is a high possibility that the communication infrastructure is interrupted; therefore, modeling information asynchrony can contribute to humanitarian logistics.

Furthermore, after a disaster, where there is urgency and a lack of information, conventional numerical methods cannot be used effectively because they need much input

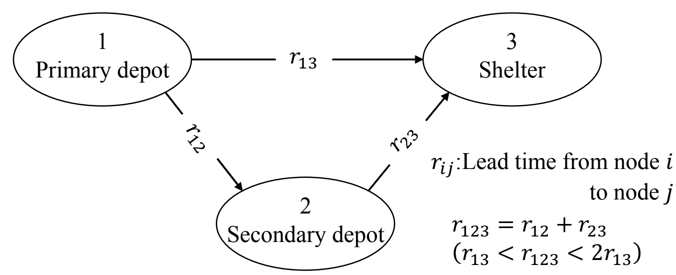

Fig. 2. Supply chain network

and heuristic techniques which require a lot of time to solve. Analytic models with closed-form solution that can be solved easily without much information are the practical alternative to numerical methods. This approach can clarify the properties of solutions close to the global optimum and contribute to the formulation of guidelines for humanitarian logistics.

This research develops an inventory distribution model to clarify the mathematical properties of the optimal control strategy. Kawase et al. (2018) formulated this problem as a deterministic optimal control problem, which is one of the proposed analytic models. We make this formulation stochastic based on Meng and Shen (2010) and analyze the properties when considering demand uncertainty after a disaster. Additionally, the Bayesian updating process models a learning process of uncertain information after a disaster. In this process, information asynchrony is discribed by updating at different intervals among depots. This proposed model can analyze mathematical properties of the new push- and pull-mode support including "Direct Supply," "Abolition of the secondary depot," and "Transmission system without ICT."

The rest of this paper is organized as follows: Section 2 develops our inventory distribution model consisting of the stochastic optimal control problem and the information updating algorithm. Section 3 analyzes the optimal control strategy to clarify the properties of the push-mode support. Section 4 provides numerical results and discusses the pull-mode support. The paper ends with section 5, a summary of key findings.

2. INVENTORY DISTRIBUTION MODEL

2.1 Stochastic Optimal Control Problem

Network and Object Function We assume that the SC network is a directed graph $G(N, A)$ with three nodes and three links (Fig.2), where N is a set of nodes and A is a set of arcs. Also, N^+, N^-, C_i and P_j indicate respective sets of nodes with child nodes, nodes with parent nodes, child nodes of node $i \in N^+$, and parent nodes of node $j \in N^-$. In the SC network in Fig.2, N^+ represents a set of depots. Each node represents aggregated primary depots, secondary depots and shelters in a general SC network.

We define the objective function as the summation of $TC_{IN_l}(t)$; the subjective net inventory holding cost of node $l \in N$ at the shelter, $TC_{I_i}(t)$; the inventory holding cost at depot node $i \in N^+$ and $TC_{S_l}(t)$; the inventory handling cost at the destination of node $l \in N^+$ at time $t \in [0, T]$. Letting $IN_l(t)$ be the amount of subjective net inventories of node l at the shelter, $I_i(t)$ be the amount of inventories at depot node i, $S_{ij}(t)$ be the amount of supplies per time (throughput) at link $(i,j) \in A$, and $D_l(t)$ be the amount of instantaneous subjective demand of node l at the shelter. The cost functions are as follows:

$$TC_{IN_l}(t) = h_l\Big(f_I\left([IN_l(t)]^+\right) + f_B\left([IN_l(t)]^-\right)\Big) \quad \forall l \in N, \quad (1)$$

$$h_l = \begin{cases} h'_l - h'_{l-1} & l \in N^+ \\ h'_{IN} - h'_2 & l \notin N^+ \end{cases}, \; h'_{IN} = \begin{cases} h'_3 & IN_3(t) \geq 0 \\ b & IN_3(t) < 0 \end{cases} \quad \forall l \in N, \quad (2)$$

$$TC_{I_i}(t) = h'_i f_I\Big(I_i(t)\Big) \quad \forall i \in N^+, \quad (3)$$

$$TC_{S_l}(t) = \sum_{j \in C_l} cf_S\Big(\left|TS_{lj}(t)/|P_j| - S_{lj}(t - r_{lj})\right|\Big) \quad \forall l \in N^+, \quad (4)$$

$$TS_{lj}(t) = \begin{cases} \sum_{i \in C_j} S_{ji}(t) & j \in N^+ \\ D_l(t) & j \notin N^+ \end{cases} \quad \forall l \in N^+, \; j \in C_l, \quad (5)$$

$$df_X(x)/dx > 0, \; d^2 f_X(x)/dx^2 > 0,$$
$$f_X(x), df_X(x)/dx \text{ are continuous } \forall X \in [I, B, S], \quad (6)$$

where $[\cdot]^+$ represents $\max\{0,\cdot\}$ and $[\cdot]^-$ represents $-\min\{0,\cdot\}$. The parameters h'_l, b, c are, respectively, the weight of the inventory holding cost at node l, the penalty cost for falling short in supply, and the inventory handling cost. The inventory holding cost of external supplies ($l = 0$) is assumed to be 0, i.e. $h_0 = 0$. Moreover, the penalty cost is higher than the inventory holding cost, which increases as the relief goods get closer to the shelter ($h'_l < h'_{l+1} < b$). Eq.(4) shows the changes in inventories and Eq.(5) shows the outflows at destination j of depot node l.

Inventory Dynamics The dynamics of net inventory $IN_l(t)$, inventory $I_i(t)$, and demand $D_l(t)$ are as follows:

$$dIN_l(t) = \left[\sum_{i \in P_3} S_{i3}(t - r_{i3}) - D_l(t)\right] dt \quad IN_l(0) < 0 \quad \forall l \in N, \quad (7)$$

$$D_l(t)\,dt = \overline{D}_l(t)\,dt + D_l^{SD}(t)\,dz_l(t) \quad \forall l \in N, \quad (8)$$

$$IN_3(t) = IN_2(t), \quad (9)$$

$$\dot{I}_1(t) = 0, \quad I_1(0) = 0, \quad (10)$$

$$\dot{I}_2(t) = S_{12}(t - r_{12}) - S_{23}(t), \quad I_2(0) \geq 0, \quad (11)$$

where $z_l(t)$ depicts the standard wiener process, and $\overline{D}_l(t)$ and $D_l^{SD}(t)$ depict parameters representing the mean value and standard deviation of the subjective demand $D_l(t)$, respectively. The function $D_l(t)\,dt$ follows the normal distribution $N(\overline{D}_l(t)\,dt, (D_l^{SD}(t))^2 dt)$. In section 2.2, we will describe our information updating algorithm for $D_l(t)$. Eq.(7) shows the dynamics of the subjective net stock inventory which consist of inflows into the shelter and subjective demand. Moreover, we define $IN_3(t)$, which is the true value, as Eq.(9) for convenience. By contrast, we assume that the information regarding depots is perfect, letting the dynamics of inventory at depots be Eqs.(10)(11). From Eq.(10), the amount of inventory at the primary depot is explicitly 0 (i.e. $I_1^*(t) = 0$; the primary depot is assumed to be the transfer center). For simplicity, $I_2(t)$ is referred to as $I(t)$.

Optimization Problem Giving the initial condition of $S_{ij}(t)$ to Eqs.(1)-(11), the inventory distribution problem is formulated as a stochastic optimal control problem with $IN_l(t)$ and $I_i(t)$ as state variables and $S_{ij}(t)$ as control variables.

$$\min V = E\int_0^T \Big[\sum_{l \in N} TC_{IN_l}(t) + \sum_{i \in N^+} TC_{I_i}(t) + \sum_{l \in N^+} TC_{S_l}(t)\Big] dt, \quad (12)$$

subject to Eqs.(1) − (11) and

$$S_{ij}(t) = 0 \quad \forall t \in [-r_{ij}, 0), \; j \in C_i, \; i \in N^+. \quad (13)$$

In fact, the inventory $I_i(t)$ and the throughput $S_{ij}(t)$ are nonnegative but in this problem constraints are explicitly not included, because numerical calculation is necessary to solve the stochastic differential equations of Eq.(7). $S_{ij}(t)$ is also constrained because not only shelters but also depots have damages after a disaster. In this problem, $S_{ij}(t)$ is constrained by minimizing changes in inventories shown in Eq.(4). The integrand of Eq.(12) is narrowly convex with respect to state variables and control variables (because $f_X(x)$ gradually increases), and the differential equations of Eqs.(7)-(11) are linear with respect to control variables; therefore, the stochastic optimal control problem shown in Eqs.(1)-(13) satisfies Mangasarian Sufficient Conditions (Mangasarian (1966)). Accordingly, the optimal solution derived by the maximum principle, which is the necessary condition for the extremum of the dynamic optimization problem, is the only global optimal solution.

2.2 Information Updating Algorithm

Bayesian Updating This section shows our information updating algorithm for information asynchrony. Considering information uncertainty after a disaster, accumulation of much information should be modeled. The Bayesian updating process can describe such a learning process.

First, the probability distribution (prior distribution) of the mean value $\mu_{l,n-1}(t)$ of the subjective demand distribution, which has already been updated $n-1$ times at time t, is updated to the posterior distribution using the information regarding demand $\tilde{D}(t)$ from the shelter. When each distribution is normal, the n-th updating of the probability distribution (posterior distribution) of the mean value $\mu_{ln}(t)$ of the subjective demand distribution is formulated as follows:

$$\overline{\mu}_{ln}(t) = \frac{(\sigma(t))^2}{(\sigma(t))^2 + \left(\mu_{l,n-1}^{SD}(t)\right)^2}\overline{\mu}_{l,n-1}(t) + \frac{\left(\mu_{l,n-1}^{SD}(t)\right)^2}{(\sigma(t))^2 + \left(\mu_{l,n-1}^{SD}(t)\right)^2}\tilde{D}(t), \quad (14)$$

$$\left(\mu_{ln}^{SD}(t)\right)^2 = \frac{(\sigma(t))^2\left(\mu_{l,n-1}^{SD}(t)\right)^2}{(\sigma(t))^2 + \left(\mu_{l,n-1}^{SD}(t)\right)^2}, \quad (15)$$

$$\overline{\mu}_{l0}(t) = \overline{D}(t), \quad \mu_{l0}^{SD}(t) = D^{SD}(t), \tag{16}$$

where $\sigma(t)$ is the standard deviation of the likelihood distribution (the conditional probability distribution to obtain $\tilde{D}(t)$ when depot l predicts as $\mu_{l,n-1}(t)$), $\overline{\mu}_{ln}(t)$ and $\mu_{ln}^{SD}(t)$ are the mean value and standard deviation, respectively, of the posterior distribution in the n-th update. Moreover, parameters $\overline{D}(t)$ and $D^{SD}(t)$, respectively, represent the mean value and standard deviation of the predicted demand $D(t)$ at the shelter.

Next, subjective demand $D_{ln}(t)$ is predicted in the n-th update using the posterior distribution. When each distribution is normal, the probability distribution (predicted distribution) of $D_{ln}(t)$ is updated as follows:

$$\overline{D}_{ln}(t) = \overline{\mu}_{ln}(t), \tag{17}$$

$$\left(D_{ln}^{SD}(t)\right)^2 = \left(\mu_{ln}^{SD}(t)\right)^2 + (\sigma(t))^2, \tag{18}$$

where $\overline{D}_{ln}(t)$ and $D_{ln}^{SD}(t)$, respectively, indicate the mean value and standard deviation of the predicted distribution. Since subjective demand follows a normal distribution according to Eq.(8), it can be updated using Eqs.(14)-(18). Considering that demand is unsteady after a disaster, it is necessary to predict the demand until the next update, if the information updating is occasional. We define the mean value of each distribution as follows:

$$\overline{D}_{ln}(t) = -\frac{\overline{D}_{ln}(nk_l)}{T - nk_l}(t - T), \quad t \in [nk_l, nk_l + k_l), \tag{19}$$

$$\overline{\mu}_{ln}(t) = \overline{D}_{ln}(t), \qquad n = 0, 1, ..., \lfloor T/k_i \rfloor - 1, \tag{20}$$

and standard deviation as a constant until the next update. When information is updated at k_l intervals, we have,

$$\overline{D}_l(t) = \overline{D}_{ln}(t), \qquad t \in [nk_l, nk_l + k_l), \tag{21}$$

$$D_l^{SD}(t) = D_{ln}^{SD}(t), \qquad n = 0, 1, ..., \lfloor T/k_l \rfloor - 1, \tag{22}$$

where update intervals k_l become smaller as depot l is closer to the shelter ($k_1 > k_2$) because information is signaled step by step as in the Kanban System. In this research, we analyze the pull-mode support with subjective demand $D_l(t)$ updated using this information updating algorithm. By contrast, for the push-mode support, predicted demand $D(t)$ is not updated ($D_1(t) = D_2(t) = D(t)$) because of transportation without a request from the shelter. Therefore, the subjective net inventories are always equivalent ($IN_1(t) = IN_2(t) = IN(t)$). In other words, the push- and pull-mode are modeled only depending on whether this information updating algorithm is applied or not, which means that they can be evaluated using the same objective function Eq.(12).

The Bullwhip Effect This section shows that our information updating algorithm can model the bullwhip effect. For simplicity, we assume demand to be steady. In business logistics, the bullwhip effect is quantitatively evaluated using the value obtained by dividing the variance of the order quantity by that of the demand. Based on this, we use the variance ratio of the subjective demand. Since the number of information updating n decreases in the upstream of the SC network (because $k_1 > k_2$), $(D_{l,n-1}^{SD})^2/(D_{ln}^{SD})^2 > 1$ implies the bullwhip effect. Additionally, since σ does not depend on n, from Eq.(18) $(\mu_{l,n-1}^{SD})^2/(\mu_{ln}^{SD})^2 > 1$ also implies the bullwhip effect. Letting $\nu = \sigma^{-2}$, $\nu_{ln} = (\mu_{ln}^{SD})^{-2}$, we obtain,

$$\nu_{ln} = \nu_{l,n-1} + \nu, \tag{23}$$

$$\frac{(\mu_{l,n-1}^{SD})^2}{(\mu_{ln}^{SD})^2} = \frac{\nu_{ln}}{\nu_{l,n-1}} = \frac{\nu_{l,n-1} + \nu}{\nu_{l,n-1}} > 1. \tag{24}$$

3. ANALYSIS OF OPTIMAL CONTROL

In this section, we derive optimal solutions of the stochastic optimal control problem and clarify the properties of the push-mode support. Functions $f_I(x), f_B(x), f_S(x)$ are defined as follows:

$$f_I(x) = f_B(x) = f_S(x) = x^\alpha, \quad \alpha > 1, \tag{25}$$

where minimizing the objective function (12) is equivalent to maximizing the cost disorder, because $\alpha > 1$. We let $\alpha = 2$ to derive the optimal solution explicitly. Furthermore, the following equations are assumed.

Assumption To derive the optimal solution explicitly from the stochastic optimal control problem shown in Eqs.(1)-(13), the assumptions are as follows:

(1) Let T be the time when demand becomes 0, $\overline{D}_l(T) = 0$.
(2) Demand decreases constantly over time, $d\overline{D}_l(t)/dt = \dot{\overline{D}}_l < 0$.
(3) The inventory holding cost at the shelter is twice that at the primary depot, $h'_{IN} = 2h'_1$.
(4) The secondary depot is not ready after a disaster, $S_{23}(t) = 0, \ \forall t \in [0, r_{12})$.

Assumptions (1) and (2) allow the optimal solution to be expressed in closed-form as possible, making in readily interpretable. To solve the stochastic differential equations called adjoint equations shown in section 3.1, assumptiona (3) and (4) are necessary, however their detailed solution process is omitted because of space limitations. Assumptions (1)-(3) are realistic. However, Assumption (4) has limitations in that the secondary depot cannot transport relief goods to the shelter immediately after the disaster.

3.1 Optimality Conditions

The optimality conditions of the optimization problem shown in Eqs.(1)-(13) are obtained using the maximum principle as follows:

$$S_{ij}^*(t) = \arg\min_{S_{ij}} \{H(t) + \chi_{[0,T-r_{ij}]}(t)[H(t + r_{ij})]\}$$
$$\forall j \in C_i, \ i \in N^+, \tag{26}$$

$$d\lambda_{IN_l}(t) = -2h_l IN_l(t)\,dt + \Lambda_{IN_l}(t)\,dz_l(t),$$
$$\lambda_{IN_l}(T) = 0, \quad \forall l \in N^+, \tag{27}$$

$$\dot{\lambda}_I(t) = -2h'_2 I(t), \ \lambda_I(T) = 0, \tag{28}$$

$$H(t) = F(t) + \sum_{l \in N^+} \lambda_{IN_l}(t)\left[\sum_{i \in P_3} S_{i3}(t - r_{i3}) - \overline{D}_l(t)\right] + \lambda_I(t)(S_{12}(t - r_{12}) - S_{23}(t)) - \sum_{l \in N^+} \Lambda_{IN_l}(t) D_l^{SD}(t), \tag{29}$$

$$F(t) = \sum_{l\in N} TC_{IN_l}(t) + \sum_{i\in N^+} TC_{I_i}(t) + \sum_{l\in N^+} TC_{S_l}(t), \quad (30)$$

and state equations (7) – (11),

and initial condition (13),

where $\lambda_{IN_l}(t), \lambda_I(t), \Lambda_{IN_l}(t)$, and $\Lambda_I(t)$ are adjoint variables, and $\chi_{[0,T-r_{ij}]}(t)$ is a binary variable taking the value of 1 if it is $t \in [0, T - r_{ij}]$ and 0 otherwise.

3.2 Optimal Control Strategy

From the optimality conditions, the optimal solutions $(IN^*(t), I^*(t), S^*_{ij}(t))$ are as follows (however, a part of them and their detailed derivation process is omitted because of space limitations):

$$I^*(t) = \begin{cases} I(0), & t \in (0, r_{12}], \\ I(0)\exp\left(-t\sqrt{\frac{h'_2}{c}}\right)\frac{1+y_I^2(t)}{1+y_I^2(r_{12})}, & t \in (r_{12}, T], \end{cases} \quad (31)$$

$$IN^*(t) = U(t)\left[IN^*(r_{123}) - \int_{r_{123}}^{t}\frac{D^{SD}(s)}{U(s)}dz(s)\right], \quad t \in (r_{123}, T], \quad (32)$$

$$S^*_{12}(t) = -\sqrt{\frac{h'_2}{c}}\frac{1-y_I^2(t+r_{12})}{1+y_I^2(t+r_{12})}I^*(t+r_{12}) + S^*_{23}(t+r_{12}), \quad t \in [0, T-r_{12}), \quad (33)$$

$$S^*_{23}(t) = \hat{S}(t+r_{23}), \quad t \in [r_{12}, T-r_{23}), \quad (34)$$

$$S^*_{13}(t) = \hat{S}(t+r_{13}), \quad t \in [0, T-r_{13}), \quad (35)$$

$$\hat{S}(t) = -\exp\left[(r_{123}-t)\sqrt{\frac{2h'_{IN}}{c}}\right]\sqrt{\frac{h'_{IN}}{2c}}\frac{1-y_{IN}^2(t)}{1+y_{IN}^2(r_{123})} \times\left[IN^*(r_{123}) - \int_{r_{123}}^{t}\frac{D^{SD}(s)}{U(s)}dz(s)\right] + \frac{\overline{D}(t)}{2}, \quad t \in [r_{123}, T], \quad (36)$$

where variables are as follows:

$$U(t) = \begin{cases} \exp\left[(r_{13}-t)\sqrt{\frac{h'_{IN}}{c}}\right]\frac{\psi^+ - \psi^- y_{IN}^2(t)}{\psi^+ - \psi^- y_{IN}^2(r_{13})}, & t \in (r_{13}, r_{123}], \\ \exp\left[(r_{123}-t)\sqrt{\frac{2h'_{IN}}{c}}\right]\frac{1+y_{IN}^2(t)}{1+y_{IN}^2(r_{123})}, & t \in (r_{123}, T], \end{cases} \quad (37)$$

$$y_I(t) = \exp\left((t-T)\sqrt{\frac{h'_2}{c}}\right), \quad t \in [r_{12}, T], \quad (38)$$

$$y_{IN}(t) = \begin{cases} \exp\left[(t-r_{123})\sqrt{\frac{h'_{IN}}{c}}\right], & t \in [r_{13}, r_{123}), \\ \exp\left[(t-T)\sqrt{\frac{2h'_{IN}}{c}}\right], & t \in [r_{123}, T), \end{cases} \quad (39)$$

$$\psi^{\pm} = 2\sqrt{ch'_{IN}}\left(\frac{1-y_{IN}^2(r_{123})}{1+y_{IN}^2(r_{123})} \pm 1\right), \quad (40)$$

where $0 < y_I(t),\ y_{IN}(t) \le 1,\ \psi^- < 0$.

At first, we analyze the optimal dynamics of inventory at the secondary depot and prove the following result:

Theorem 1. There is no need to pre-store at the secondary depot and to add stock after a disaster, that is, $I^*(t) = 0\ \forall t \in [0, T]$.

Proof. Although the optimal amount of inventories is clearly $I^*(t) > 0$ from the Eqs.(11)(31), the derivative of $I^*(t)$ is,

$$\dot{I}^*(t) = -I(0)\exp\left(-t\sqrt{\frac{h'_2}{c}}\right)\frac{1-y_I(t)}{1+y_I(r_{12})} < 0, \quad (41)$$

and the limit of $I^*(T)$ is,

$$\lim_{T\to\infty} I^*(T) = I(0)\cdot 0 \cdot \frac{2}{1+0} = 0, \quad (42)$$

hence, $I^*(t)$ asymptotically approaches 0 from the initial value $I(0) \ge 0$. Actually, the initial value $I(0)$, that is the amount of pre-stocks, is also a variable. Then, we solve the optimal amount of pre-stocks $I^*(0)$ which minimizes the objective function shown in Eq.(12). From the optimal solutions Eq.(31)-(40), the first derivative of the optimal objective function $V^* = \min V$ is as follows:

$$\frac{\partial V^*}{\partial I(0)} = 2h'_2 I(0)\left[r_{12} + \int_{r_{12}}^{T}\exp\left(-2t\sqrt{\frac{h'_2}{c}}\right) \times\frac{1+y_{IN}^4(t)}{(1+y_{IN}^2(r_{12}))^2}dt\right]. \quad (43)$$

Because $[\cdot]$ in Eq.(43) is obviously nonnegative, $I^*(0) = 0$, the optimal amount of inventory at the secondary depot, is $I^*(t) = 0\ \forall t \in [0, T]$.

Next, we analyze the optimal control path and clarify that "Direct Supply" is effective. Specifically, we prove the following result:

Theorem 2. $E[S^*_{13}(t)] > E[S^*_{12}(t)] = E[S^*_{23}(t+r_{12})]\ \forall t \in [r_{123} - r_{13}, T - r_{123})$.

Proof. From Eqs.(33)-(36) and *Theorem 1*, throughputs are as follows:

$$S^*_{12}(t) = \hat{S}(t+r_{123}), \quad (44)$$

$$S^*_{23}(t) = S^*_{12}(t-r_{12}), \quad (45)$$

$$S^*_{13}(t) = \hat{S}(t+r_{13}), \quad (46)$$

where $S^*_{12}(t) < S^*_{13}(t)$ when $\hat{S}(t)$ decreases narrowly, because $r_{13} < r_{123}$. However, the wiener process $z(t)$ included in $\hat{S}(t)$ makes it impossible to directly differentiate $\hat{S}(t)$. Then the expected value of $\hat{S}(t)$,

$$E\left[\hat{S}(t)\right] = -\exp\left[(r_{123}-t)\sqrt{\frac{2h'_{IN}}{c}}\right]\sqrt{\frac{h'_{IN}}{2c}} \times\frac{1-y_{IN}^2(t)}{1+y_{IN}^2(r_{123})}E\left[IN^*(r_{123})\right] + \frac{\overline{D}(t)}{2}, \quad (47)$$

is differentiated as follows:

$$dE\left[\hat{S}(t)\right]/dt = \exp\left[(r_{123}-t)\sqrt{\frac{2h'_{IN}}{c}}\right]\frac{h'_{IN}}{c} \times \frac{1+y_{IN}^2(t)}{1+y_{IN}^2(r_{123})}E\left[IN^*(r_{123})\right] + \frac{\dot{\overline{D}}}{2}. \quad (48)$$

From Eqs.(7)(13)(35) and Assumption (4), we obtain,

$$\begin{aligned} &E\left[IN^*(r_{123})\right] \\ &= U(r_{123})\left[E\left[IN^*(r_{13})\right] - \frac{c\dot{\overline{D}}}{2h'_{IN}}\right] + c\Big[\frac{\dot{\overline{D}}}{2h'_{IN}} \\ &\quad - \Big(\overline{D}(r_{123}) + \frac{\dot{\overline{D}}\psi_{IN}(r_{123})}{2h'_{IN}}\Big)\frac{1-y_{IN}^2(r_{13})}{\psi^+ - \psi^- y_{IN}^2(r_{13})}\Big] \\ &= U(r_{123})E\left[IN^*(r_{13})\right] - c\overline{D}(r_{123})\frac{1-y_{IN}^2(r_{13})}{\psi^+ - \psi^- y_{IN}^2(r_{13})} \\ &\quad - \frac{c\dot{\overline{D}}}{2h'_{IN}}\Big[U(r_{123}) - 1 + \psi_{IN}(r_{123})\frac{1-y_{IN}^2(r_{13})}{\psi^+ - \psi^- y_{IN}^2(r_{13})}\Big], \quad (49) \end{aligned}$$

where $[\cdot]$ in Eq.(49) is,

$$\begin{aligned} &U(r_{123}) - 1 + \psi_{IN}(r_{123})\frac{1-y_{IN}^2(r_{13})}{\psi^+ - \psi^- y_{IN}^2(r_{13})} \\ &= y_{IN}(r_{13})\frac{4\sqrt{ch'_{IN}}}{\psi^+ - \psi^- y_{IN}^2(r_{13})} - \frac{4\sqrt{ch'_{IN}}}{\psi^+ - \psi^- y_{IN}^2(r_{13})} \\ &\qquad + 2\sqrt{ch'_{IN}}\frac{1-y_{IN}^2(r_{13})}{\psi^+ - \psi^- y_{IN}^2(r_{13})} \\ &= -\frac{2\sqrt{ch'_{IN}}}{\psi^+ - \psi^- y_{IN}^2(r_{13})}\left(y_{IN}(r_{13}) - 1\right)^2 < 0. \quad (50) \end{aligned}$$

From Eqs.(49)(50) and Assumption (2), we obtain $E\left[IN^*(r_{123})\right] < 0$ and

$$\frac{dE\left[\hat{S}(t)\right]}{dt} < 0. \quad (51)$$

This completes the proof.

The parameter $\alpha(=2)$ indicates a weight of cost disorder, which provides an optimal control without deviation, that is $S_{12}^*(t) \neq 0$. In other words, when $\alpha \to 1$, the optimal control can be expected to involve only direct supply. From *Theorem 1*, there is no need to add stock at the secondary depot, and from *Theorem 2*, the optimal control can be expected to involve only direct supply; therefore, it is clear that "Abolition of the secondary depot" is effective.

Finally, we show the dynamic property of the optimal net inventory at the shelter.

Theorem 3. Maintaining positive inventory at the shelter is the optimal strategy when demand is uncertain and the penalty cost for falling short in supply is sufficiently high, that is, $IN^*(t) \geq 0$, if $D^{SD}(t) \neq 0$ and $b \to \infty$.

Proof. From Eq.(37), we obtain $\lim_{T\to\infty} U(T) = 0$. Then the long-term expected value $\mu^\infty = \lim_{T\to\infty} E\left[IN^*(T)\right]$ is as follows:

$$\mu^\infty = \lim_{T\to\infty} E\left[IN^*(T)\right] = 0 \cdot E\left[IN^*(r_{123})\right] = 0. \quad (52)$$

Also, from Eqs.(7)(32)(34)(35)(36)(52), we obtain the dynamics of $IN^*(t)$,

$$dIN^*(t) = -\sqrt{\frac{2h'_{IN}}{c}}\frac{1-y_{IN}^2(t)}{1+y_{IN}^2(t)}\left[IN^*(t) - \mu^\infty\right]dt - D^{SD}(t)\,dz(t), \quad (53)$$

where $\sqrt{\frac{2h'_{IN}}{c}\frac{1-y_{IN}^2(t)}{1+y_{IN}^2(t)}} = v \geq 0$; thus, Eq.(53) means that

$$dIN^*(t)\begin{cases} <0 & IN^*(t) > \mu^\infty, \\ =0 & IN^*(t) = \mu^\infty, \\ >0 & IN^*(t) < \mu^\infty. \end{cases} \quad (54)$$

This stochastic differential equation is called the Ornstein-Uhlenbeck process (Uhlenbeck and Ornstein (1930)). Then we focus on the regression speed v. From Eq.(2), the parameter h'_{IN} is

$$h'_{IN} = \begin{cases} h'_3 & IN(t) \geq 0 \\ b & IN(t) < 0 \end{cases}, \quad (55)$$

where $h'_3 < b$. After a disaster, the penalty cost for falling short in supply is considered to be sufficiently high, that is $b \to \infty$ and we obtain $|v| \to \infty$. In other words, if supplies are short $(IN^*(t) < 0)$ $IN^*(t)$ momentarily returns to $\mu^\infty(=0)$, and otherwise $(IN^*(t) \geq 0)$ it returns at speed $|v| < \infty$, which means that the optimal strategy is to maintain positive inventory at the shelter. This inventory strategy is optimal only when considering demand uncertainty because when $D^{SD}(t) = 0$, from Eqs.(53)(54), $IN^*(t)$ only approaches $\mu^\infty(=0)$ from the initial value $IN(0) < 0$.

Table 1. Parameter settings

t	$[0, 10)$	$[10, 20)$	$[20, 30]$
$\tilde{D}(t)$	4	2	1
$\overline{D}(t)$	$-0.15(t-T)$		
$\{D^{SD}(t), \sigma(t)\}$	$\{100, 50\}$		
$\{r_{12}, r_{23}, r_{13}\}$	$\{h'_2, h'_3, b\}$	c	$IN(0)$
$\{3, 2, 4\}$	$\{0.5, 0.7, 1\}$	10	-200

4. NUMERICAL EXAMPLE

4.1 Settings

This section clarifies the property of the pull-mode support under information asynchrony, comparing the objective functions of the push- and pull-mode which are calculated by the Monte Carlo simulation. The parameters are set in Table.1, and two cases, called case-a and case-b, are examined. Case-a is a situation where depots can share prediction errors $z_l(t)$, that is $R[z_1(t), z_2(t)] = 1$. By contrast, case-b is a situation where each depot has different prediction errors, $R[z_1(t), z_2(t)] \neq 1$. The stochastic differentiation $dz_l(t)$ of $z_l(t)$ is calculated using the Euler-Maruyama method (Maruyama (1955)).

4.2 Results

Fig.3 shows the probability $\mathscr{P} = Prob(V^{push} > V^{pull}(k_1, k_2))$, where V^{push} is the objective function of the push-mode support and $V^{pull}(k_1, k_2)$ is that of the pull-mode support. The horizontal and vertical axis represent the information update intervals (k_1, k_2), where the number of information updating n is higher as (k_1, k_2) approaches the

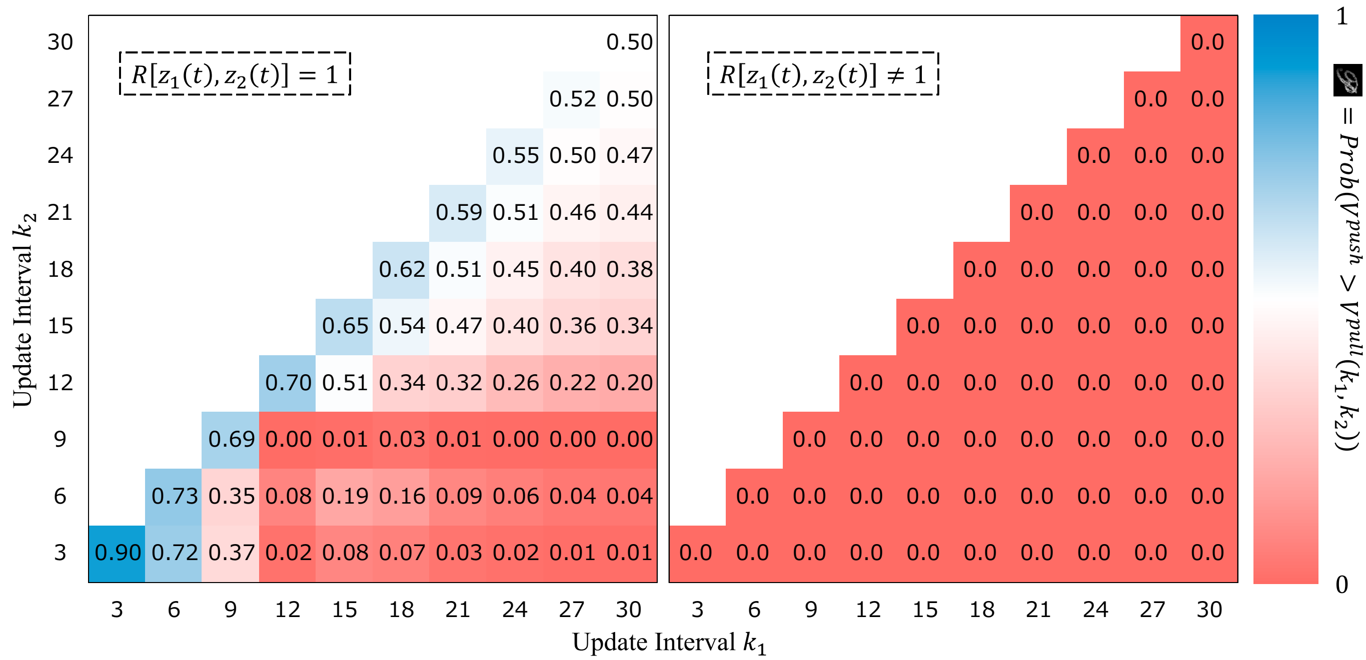

Fig. 3. Comparison of the objective functions of the push- and pull-mode support(left:case-a, right:case-b)

origin. $V^{pull}(30, 30)$ is equal to V^{push} because $(k_1, k_2) = (30, 30)$ means that n is 0. The numbers and color in the graph indicate the probability $\mathscr{P}$, where red shows that the push-mode is more effective than the pull-mode and blue shows the opposite.

The left side of Fig.3 shows that the push-mode tends to be more effective when $k_1 \neq k_2$; in addition, as $k_1 - k_2$ becomes larger, the probability $\mathscr{P}$ becomes smaller, that is, the push-mode becomes more effective. On the other hand, when $k_1 = k_2$ the pull-mode is more effective, in addition, the pull-mode becomes more effective the closer it is to the origin. Therefore, it is suggested that the pull-mode can be effective under information synchrony among the depots, which indicates that "Transmission system without ICT" may be an undesirable strategy and that the information paradox, wherein the system gets worse by using information, may occur under information asynchrony.

The right side of Fig.3 shows case-b, where the push-mode is more effective regardless of k_l. In other words, the pull-mode becomes a bad strategy in the situation where each depot has different prediction errors. In summary, the necessary conditions for the pull-mode to be effective are not only information synchrony, but also sharing of prediction errors.

5. DISCUSSION AND CONCLUSION

This research proposed the inventory distribution model to clarify the mathematical properties of empirical strategies. Our proposed model was formulated as a stochastic optimal control problem which can analyze the optimal control strategy on a simple supply chain network. As a result, it was revealed that direct supply from the primary depot to the shelter and abolition of the secondary depot are effective. Moreover, we showed that under information asynchrony the information paradox may occur.

From these results, we propose new control strategies as shown in Fig.4 and 5. For the push-mode support, relief goods are transported directly from the primary depot to the shelter rather than via the secondary depot (Fig.4). By contrast, Fig.5 shows the pull-mode proposed strategy. To satisfy the necessary conditions for the pull-mode to be more effective than the push-mode, the control tower manages the information to avoid information asynchrony

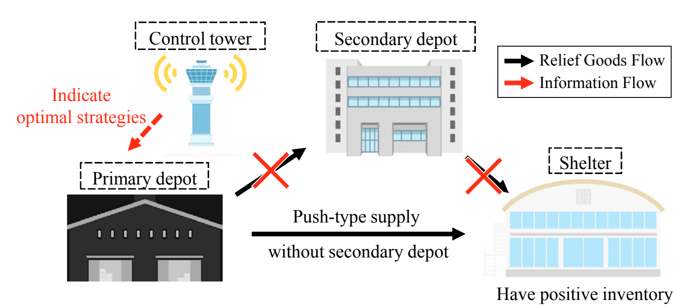

Fig. 4. Proposed control strategy (Push-mode support)

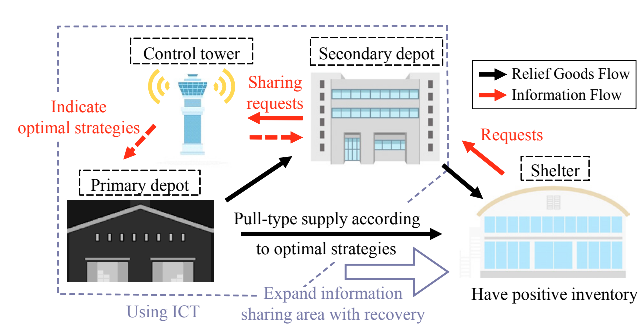

Fig. 5. Proposed control strategy (Pull-mode support)

among depots and instructs them regarding the optimal strategies to share prediction errors.

Future research should clarify the properties for a general SC network, where there are multiple depots and shelters. For efficient logistics, it is essential to consider the economies of agglomeration caused by collecting relief goods and logistics equipment. For example, when there are multiple shelters as in a general network, indirect supply (collecting relief goods at secondary depots and transporting) is expected to be more efficient than direct supply. Thus, analyzing a general SC network may reveal different properties from this research. Although our concerns in the analysis of a general network are competitive throughputs with the same origin/destination, independent properties of optimal throughputs from inventories at origin depots have an implication for the applicability of our proposed model to the analysis of a general network.

Additionally, to improve our information updating algorithm is a more challenging task. In this research, the Bayesian updating process was applied to model information asynchrony but the value or reliability of information cannot be considered. After a disaster there are many kinds of information and their reliability is diverse. In order to solve this problem, information priorities and selection systems such as demand forecasting using data fusion as in Sheu (2010) are necessary. Subjective demand depends greatly on the value of information; hence, future research can improve the information updating algorithm.

ACKNOWLEDGEMENTS

We would like to thank Editage (www.editage.jp) for English language editing.

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